Thermodynamics and Statistical Physics

Part I – Thermodynamics

Resit

Wednesday, April 12 2023, 15:00-17:00, Aletta Jacobshal

The total number of points that can be reached in this exam is 90.

Final grade = (points/10) + 1.

PROBLEM 1

Score: a+b+c =10+10+10=30

1) The pressure in in the thermosphere outside the international space station (about 340 km above sea level) is about 10^{-5} Pa and the temperature is about 1100 °C. Estimate the mean free path of an N_2 molecule (M_{N2}=28 g/mol, d=0.37 nm) under these conditions **(10 pt).**
2) Express the first law of thermodynamics as an equation and with words. **(10 pt)**
3) Explain the concept of thermodynamic "reversibility" for the case of an expanding ideal gas. **(10 pt)**

PROBLEM 2

Score: a+b+c= 12+12+6=30

Your apartment had a very old window of area $A = 1.5$ m² with thermal conductance $\frac{\kappa}{L} = 6$ $\mathrm{Wm^{-2}K^{-1}}$. This window is replaced by a state of the art double-pane window with $\frac{\kappa}{L} = 1$ $\mathrm{Wm^{-2}K^{-1}}$.

a) Use the thermal diffusion equation in one dimension

$$\frac{\partial T}{\partial t} = D\frac{\partial^2 T}{\partial x^2}$$

To determine a functional form of the temperature profile through the window $T(x)$, with the temperatures T_{outside} and T_{inside} both constant. **(10 pt).**

b) Give a functional form for the heat flux through the window using $T(x)$. **(10 pt).**
c) How much energy do you save per winter day (24 hours), assuming a constant temperature outside of -5 °C and inside of 20 °C? **(10 pt).**

PROBLEM 3

Score: a+b+c+d =8+8+8+6 =30

A Carnot cycle describes a hypothetical engine which connects two reservoirs and undergoes the following *reversible* processes:

i. isothermal expansion at temperature T_h
ii. adiabatic expansion to temperature T_c
iii. isothermal compression at temperature T_c
iv. adiabatic compression to the initial state

During step *i*, a quantity of heat Q_h flows from the reservoir at T_h into the engine. During step *iii*, a quantity of heat Q_c flows from the engine into the reservoir at T_c.

a) Sketch the thermodynamic cycle in the usual way (p,V)-diagram. Clearly indicate the direction of the cycle. Indicate where and in which direction the engine exchanges heat and/or work with the outside world. **(8 pt)**

b) Give the entropy change per cycle of both reservoirs in terms of the given quantities. **(8 pt)**

c) Assume that the entropy change in the hot reservoir is exactly compensated by the entropy change in the cold reservoir. Show, that the efficiency of the cycle equals $\eta = 1 - T_c/T_h$. **(8 pt)**

d) The coal power plant in Eemshaven produces about 12 TWh of electricity per year from about 3 million tons of coal (thermal energy content of coal: 9 kWh kg^{-1}). Determine the efficiency of the power plant and estimate T_c and T_h assuming the power plant is an idealized Carnot engine. Is the result for T_h realistic? **(6 pt)**

Constants:

Avogardro's number:	N_A=6.02 10^{23} mol^{-1}
Boltzmann constant:	k_B=1.381 10^{-23} J\K
Gas constant:	R=8.31 $JK^{-1}mol^{-1}$
Atomic mass unit (u):	m_u=1.67 10^{-27} kg
Electronvolt:	1 eV=1.6 10^{-19} J